Student Workbook

Inverse Functions

Name:

Date:

Period:

Learning Target: I can determine whether a function has an inverse, find inverse functions algebraically, and verify two functions are inverses.

Vocabulary

Term	Meaning
Inverse Function, $f^{-1}(x)$	A function that "undoes" $f(x)$: if $f(a) = b$, then $f^{-1}(b) = a$.
One-to-One Function	A function where every output corresponds to exactly one input. Only these have inverse FUNCTIONS.
Horizontal Line Test	If any horizontal line crosses the graph more than once, the function is NOT one-to-one and has no inverse function.

COMMON MISTAKE

$f^{-1}(x)$ does NOT mean $1/f(x)$! The "-1" here is inverse-function notation, not an exponent. These are usually completely different expressions.

Key Concept: Finding an Inverse Algebraically

METHOD

1. Replace $f(x)$ with y.
2. Swap x and y.
3. Solve the new equation for y.
4. Replace y with $f^{-1}(x)$.

EXAMPLE 1

Find the inverse of $f(x) = 3x - 4$.

$y = 3x - 4 \rightarrow$ swap: $x = 3y - 4$

Solve for y: $3y = x + 4 \rightarrow y =$ ____________________

$f^{-1}(x) =$ ____________________

EXAMPLE 2 — Verifying an Inverse

Verify that f and f^{-1} from Example 1 are truly inverses by checking $f(f^{-1}(x)) = x$.

$f((x+4)/3) = 3 \cdot (x+4)/3 - 4 =$ ____________

Key Concept: Inverses and Restricted Domains

WATCH OUT FOR RADICALS AND QUADRATICS

If f(x) involves a square root or a quadratic, its inverse often needs a domain restriction to make sure you're only taking the "correct half" of the relationship.

EXAMPLE 3

Find the inverse of $f(x) = \sqrt{x - 2}$, noting any domain restriction.

$y = \sqrt{x-2} \rightarrow$ swap: $x = \sqrt{y-2} \rightarrow x^2 = y - 2 \rightarrow y = x^2 + 2$

Since the range of f(x) is $y \geq 0$, the domain of $f^{-1}(x)$ must be $x \geq$ _____.

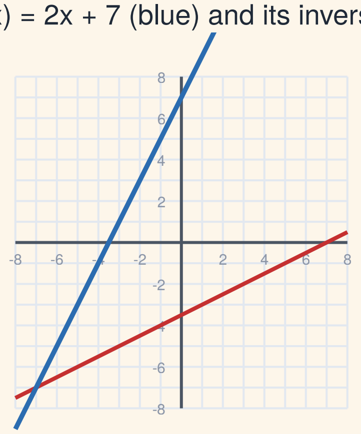
Key Concept: Graphing an Inverse

REFLECTION OVER y = x

The graph of $f^{-1}(x)$ is always the reflection of the graph of f(x) over the line y = x. If (a,b) is on f, then (b,a) is on f^{-1}.

EXAMPLE 4

$f(x) = 2x + 7$ (blue) and its inverse $f^{-1}(x) = (x-7)/2$ (red), shown as mirror images across y = x.



Guided Practice — Part A: Does It Have an Inverse Function?

A1 $f(x) = x^3$ (for all real numbers)

A2 $f(x) = x^2$ (for all real numbers)

A3 A graph is a straight, always-increasing line.

Guided Practice — Part B: Find the Inverse (Linear)

B1 $f(x) = 2x + 7$

B2 $f(x) = -4x + 1$

B3 $f(x) = (x-3)/5$

Guided Practice — Part C: Find the Inverse (With Domain Restriction)

C1 $f(x) = \sqrt{(x+4)}$

C2 $f(x) = x^2 - 1$, for $x \geq 0$

Guided Practice — Part D: Verify Two Inverses

D1 Verify that $f(x) = 2x - 6$ and $g(x) = (x+6)/2$ are inverses of each other.

BEFORE YOU MOVE ON

Check with a partner: did you swap x and y BEFORE solving for y (not after)?